

ICPM Sponsored Research

When do derivatives add value in asset allocation problems for pension funds? *

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March 7, 2013

Abstract

Recent surveys indicate that many pension fund participants aim for higher retirement income security. We investigate the added value of including derivatives in the portfolio of pension funds to achieve this goal. While modelling explicitly that the equity market exhibits both jump risk and volatility risk, we investigate the added value of including such derivatives in the asset menu of pension funds. To do so, we define preferences which incorporate specific features of pension funds but we also report the performance among several key criteria used in real life pension fund asset allocation decisions. Our results show that relatively small investments in derivatives can already achieve improvements in certainty equivalent rates of return and important other performance measures. This confirms the intuition that the use of derivatives allows pension investors to make explicit risk and return trade-offs between diffusion risk, jump risk and volatility risk and their associated risk premia.

Keywords: optimal portfolio choice, stochastic volatility and jump risks, derivatives.

*The authors gratefully acknowledge the research grant provided by the Rotman International Centre for Pension Management (ICPM) in Toronto, Canada. We would like to thank Scott Warlow, Patrick Savaria, Krishnan Chandrasekhar, Pierre Collin-Dufresne, Frank de Jong, Roderick Molenaar, Antoon Pelsser, Gabriel Petre, Berend Roorda, Jaap van Dam and participants of the ICPM Forum of October 2011 at the World Bank for their useful comments on earlier drafts of this paper.

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1 Introduction

1.1 Motivation

Due to the withdrawal of sponsors, traditional Defined Benefit (DB) systems for retirement provisions are being rapidly replaced by hybrid or Defined Contribution (DC) systems in which future retirement income is less certain. The long lasting turmoil in the financial markets since 2008 emphasizes the challenges that pension funds face in developing strategies which focus on "delivering an adequate target pension with a high degree of probability" (Blake, Cairns and Dowd (2008), Merton (2010)). In this paper we investigate whether the use of equity- and volatility-based derivative instruments may help the pension industry in meeting such challenges, by considering the design problem for optimal derivative strategies in the context of strategic asset allocation. To do so, we use asset models which incorporate risks such as stochastic volatility and the possibility of sudden price jumps. To get a clear interpretation of the end results, we use a stylized expected utility framework to capture the preference of pension funds. However, by incorporating realistic performance evaluation criteria of pension investors and by assuming that trading cannot be performed continuously, we can draw conclusions which are relevant in a practical setting.

Traditional asset allocation focuses on diversification across asset classes. We propose an allocation strategy which emphasizes diversification across risk premia (i.e., across underlying risk factors) instead. It is increasingly recognized that return variance on financial securities is stochastic and that sudden jumps in asset prices may appear. Variance uncertainty and price jumps have therefore been treated as important additional risk factors in the recent scientific literature¹ and there are significant risk premia associated with both (Carr and Wu (2009), Bollerslev, Gibson and Zhou (2011)). The extra volatility and jump risks generate market incompleteness and hence make derivatives non-redundant². Including derivatives may thus enhance investment performance, since it allows investors to diversity across risk factors and the associated risk premia. When equity and variance derivatives are added to the asset menu, investors have direct control over the separate exposures to the uncertainty in equity returns and in volatility, which facilitates an explicit trade-off between risks and the associated premiums.

The possibility to earn volatility risk premium and jump risk premium may be particularly interesting for investors with a long time horizon. Focusing on the long investment horizon, Litterman (2011) argues that pension funds may be more capable to bear volatility and jump risks than other financial institutions since volatility is mean-reverting over longer time horizons and jumps will hurt the objectives of short-term investors much more than those of long-term investors.

¹See for example Bates (1996), Broadie, Chernov and Johannes (2007), Broadie and Jain (2008), Duan and Yeh (2011), Eraker, Johannes and Polson (2003), Liu & Pan (2003), Pan (2002), and Schürhoff and Ziegler (2011).

²Redundancy of derivatives also critically depends on the possibility to trade continuously and in unrestricted quantities and the optimal asset allocation may benefit from the inclusion of derivatives if this is no longer possible. This is also the case when one introduces transaction costs or restricts oneself to trading strategies that are not allowed to vary too quickly. Therefore it is clear that derivatives are not redundant in a realistic stochastic investment environment.

Acknowledging this long term argument, we also consider the downside risk constraints of pension funds at short term horizons. In periods when the liquidity of certain assets is reduced, derivative contracts can then become valuable instruments to generate future payoffs which cannot otherwise be obtained. In that sense, the presence of stochastic jumps in asset dynamics and the absence of trading possibilities over a certain time horizon lead to similar forms of market incompleteness, which derivatives may help to mitigate. Moreover, derivatives make it possible to implement an extensive rebalancing strategy for certain assets without the requirement to trade in these assets themselves.

1.2 Main Findings

Using an extended asset menu with equity and variance derivatives, we find that the optimal portfolio not only markedly improves welfare in the expected utility framework, but also improves along most (and in some cases all) other evaluation criteria which are frequently used by pension funds. If downside risk is constrained to be no worse than the benchmark case with only stocks and bonds, the suboptimal portfolio with derivatives can still outperform the benchmark case along all evaluation criteria.

The portfolio optimally loads on equity risk premium, volatility risk premium and jump risk premium by holding a long position in equity and a short position in variance derivatives. It also contains a long position in the OTM put and a short position in the OTM call, which resembles a so-called 'collar strategy'. The portfolio loads on volatility risk via variance derivatives because these give investors a controlled exposure to this risk whereas options' sensitivity to volatility risk depends on the remaining time to maturity and the stock price path. When variance derivatives are present in the portfolio one can further enhance the retirement income security by a short position in calls and a long position in puts (but not necessarily in equal amounts). The use of shorted calls to help pay for the purchase of puts is based on the intuition that to improve the chances of achieving a desired income target in pension plans, upside potential has to be relinquished if no extra external funding is available. Timmermans, Schumacher and Ponds (2011) propose a synthetic collar position (long OTM puts and short OTM calls in equal amounts) to enhance income security for a given predefined level of downside risk. Our results indicate that this is still possible when the uncertainty in asset prices is time-varying and when their price paths may be discontinuous, as long as volatility-based derivatives can be used in the asset allocation.

The paper is organized as follows. Section 2 explains the model, the asset menu and the optimization framework. Section 3 presents the main results. Robustness checks are performed in Section 4, and we conclude by discussing some practical implications and extensions.

2 Modelling Assumptions

2.1 The Economy

We use a realistic asset model which includes stochastic volatility and price jump risk, as in Liu and Pan (2003). According to Broadie, Chernov, and Johannes (2009), such a model does incorporate the major factors driving equity option returns.

We assume that the economy contains a riskless asset which earns a constant rate of return $r > 0$ per year³ and a risky asset which is subject to both continuous price changes by diffusion and discontinuous price shocks. The single risky asset represents an equity index and although our analysis could also be performed in a multi-asset framework we will use this one single representative asset throughout the paper. The stochastic volatility process is the one introduced by Heston (1993) and features mean reversion of the equity variance process V to a long-term equilibrium level of volatility $\bar{v} > 0$. The speed of mean reversion is controlled by the parameter $\kappa > 0$ and the uncertainty in the volatility is controlled by the parameter ω , the 'volatility of volatility'. The asset dynamics are given by the stochastic differential equations

$$dB_t = rB_t dt \tag{1}$$

$$dS_t = (r + p_t) S_t dt + S_t \sqrt{V_t} dY_t + \mu S_{t-} (dN_t - \lambda V_t dt) \tag{2}$$

$$dV_t = \kappa (\bar{v} - V_t) dt + \omega \sqrt{V_t} \left(\rho dY_t + \sqrt{(1 - \rho^2)} dZ_t \right) \tag{3}$$

where B and S are the price processes for the riskless and risky asset ('Bonds' and 'Stocks'). The stochastic processes driving the dynamics are the standard Brownian Motions Y and Z and the pure jump process N which are all assumed to be independent. The model generates a correlation coefficient ρ between the price process S and the variance process V which we will assume to be negative.

The jump process N is assumed to have a jump intensity which is proportional to the variance process, as in Liu and Pan (2003), with proportionality constant $\lambda > 0$. This implements the empirical observation that in times of added market uncertainty the probability of a relatively large price change in the risky asset increases. The stochastic jump sizes μ are assumed to be fixed and downward so we take $-1 < \mu \leq 0$.

The equity risk premium p_t compensates for diffusive risk and jump risk; it takes the form $p_t = \eta V_t + \mu(\lambda - \lambda^Q) V_t$ with η controlling the reward for diffusive risk and λ^Q , the risk neutral analogue of λ , controlling the reward for jump risk. The volatility risk premium is controlled by ζ , the proportionality constant for the market price of volatility risk which enters through the stochastic pricing kernel:

³In reality interest rate risk must of course be mitigated separately (for example using interest rate swaps) but we will not incorporate that in this paper.

$$d\pi_t/\pi_{t-} = -rdt - \sqrt{V_t}(\eta dY_t + \zeta dZ_t) + \left(\frac{\lambda^Q}{\lambda} - 1\right)(dN_t - \lambda V_t dt). \quad (4)$$

This kernel and the formulas that are needed to price the derivative instruments are derived in the technical Appendix at the end of this paper. Vanilla call and put options on equity and the variance product can be priced explicitly in our model setup, as shown in Appendix. There we also show the implied volatility surface generated by our model, in Figure 2.

One can show that the model implied variance risk premium parameter in this model is $\kappa^Q - \kappa = \omega \left(\rho\eta + \sqrt{1 - \rho^2}\zeta \right)$. Thus, the variance risk premium is decomposed into two parts, one is due to the correlation with the equity diffusive risk and the other one is a compensation for the independent variance risk. Due to a negative correlation between price process S and the variance process V , and a negative value of the market price of variance risk ζ , the resulting variance risk premium parameter is negative. Christoffersen, Heston and Jacobs (2010), who consider a similar decomposition, show that the negative market price of volatility risk not only generates higher implied volatility than realized volatility, but also causes a higher variance and a higher autocorrelation of implied volatility, which explains the fatter tail of the risk neutral density relative to the realized return density.

2.2 Derivative instruments and asset menu

Apart from the basic underlying assets, stocks and bonds, we consider different possibilities for the derivatives that can be added to the portfolio. From a hedging perspective, out-of-the-money puts can, for example, be used to compensate decreasing stock prices and in particular as protection against the effects of downward jump in the risky asset. The combination of an out-of-the-money put and a shorted out-of-the-money call which was mentioned earlier may be used to transfer probability mass from the investment portfolio's rates of return for relatively extreme economic scenarios (where the stock price increases or decreases dramatically) to more moderate ones.

Since the stock process used in our economic scenario generator exhibits stochastic volatility, the return on the options that we consider not only depends on the movement of stock prices but also on the dynamics of volatility. This is the motivation to analyze the effect of adding another derivative to the asset menu as well, which has a pure exposure to volatility. We therefore introduce a variance product (i.e., the floating leg of a so-called *variance swap*) which generates a payoff which depends on the average realized variance of the risky asset during a prescribed time period (in this case, the investment horizon). As such it is purely linked to the stochastic volatility process and not directly to the asset price, although there is an indirect exposure due to the correlation between the stock price and its volatility. This will help us distinguishing between volatility risk premia and jump risk premia and thus overcome the difficulty reported in Pan (2002), where only call and put options were used, to make this distinction.

2.3 Investor preferences

We will consider the optimization of expected utility as a method to generate portfolios in which a consistent trade-off is made between risk and returns, but such portfolios must then be analyzed in terms of the more intuitive performance criteria that pension fund managers are more familiar with. We will therefore look at both in our analysis.

2.3.1 Expected utility framework

The expected utility framework is commonly used to define an objective performance measure for the optimal portfolio choice problem. For tractability and consistency with the existing literature, this will also be the approach followed in this paper. To specify the risk preferences of the pension fund we consider a utility function with constant relative risk aversion parameter γ . The structure of this utility function guarantees that the utility of terminal wealth after an investment period can be written as the product of the utility of initial wealth and the utility of the portfolio return over that period. Utility functions with constant relative risk aversion guarantee that the terminal wealth will never go negative since the absolute risk aversion will go to infinity when wealth levels approach zero. In reality, pension funds would use a higher threshold than zero for the absolute minimum of wealth they will allow in their optimization programme. We therefore use a *displaced* CRRA utility function with a threshold value which guarantees that the fund can never lose more than a percentage h (which we take to be 50%) of its value in such a period. The fact that we choose a threshold on the return over a period instead of a fixed level wealth for all periods can be interpreted as the implementation of a 'habit formation' effect. We thus formulate our optimization problem as

$$\max_{\{\alpha_i\}} E_t \left[\frac{(W_T - hW_t)^{1-\gamma}}{1-\gamma} \right] \quad (5)$$

with W_t our wealth at the initial time t and $W_T = W_t \sum_i \alpha_i \frac{A_T^i}{A_t^i}$, where A_t^i denotes the value at time t of the i -th asset in the asset menu, and where α_i is the associated portfolio weight which is constant per investment period. The asset menu includes equity, bonds, equity puts and calls and the variance product.

2.3.2 Real-life evaluation criteria

Although the expected utility framework is useful to determine an optimal investment strategy using a well-defined single goal function, additional criteria must be considered as well to draw conclusions which are relevant for pension funds.

We assume that the fund starts from a position where it has assets and liabilities⁴ which have

⁴The pension fund liabilities are deterministic in our model, given the constant interest rate assumption.

the same value, i.e. the initial funding ratio equals 100%. We consider a number of characteristics of the fund after an investment period of one year⁵ which equals the investment planning horizon for most funds. First we will look at the 2.5% quantile of the funding ratio and the expected shortfall at that level, which are also known as the Value-at-risk (VaR) and Tail-Value-at-Risk (TVaR) respectively. These measure the maximal and average level of the funding ratio in the 2.5% worst economic scenarios that are generated in our simulation and thus measure how well investment strategies are doing under such adverse scenarios. We also report the standard deviation of the rate of return as a general measure of (both upward and downward) risk. These risk measures are compared with the main statistics which consider averages over *all* scenarios, the mean rate of return and the median rate of return. Lastly, we also consider measures of income security, which is represented by the probability of reaching specific return targets. For example, we report the probability of a realized rate of return which is larger than the riskfree rate and the probability of a rate of return which is larger than the riskfree rate plus the inflation rate, which is taken as 2% annually throughout this paper. The riskfree rate plus 2% thus represents the investment target of pension fund participants in our paper.

2.4 Parameter calibration

To determine a plausible value for the risk aversion parameter γ for a typical pension fund, we calibrate it to obtain plausible stock holdings for the optimal portfolio problem without derivatives, without jump risk and without stochasticity in the volatility. This means we determine the optimal percentage to invest in stock as a function of the parameter γ and then choose this value to obtain a stock investment of 45% which roughly corresponds to the equity holding of a typical Dutch pension fund.

Parameter values are as indicated in the table below (the base case) unless stated otherwise. These parameters were chosen within the range reported in the extensive empirical literature for this model. In particular, these parameter values are based on or close to the ones reported by Pan (2002), Liu and Pan (2003), Liu, Pan and Wang (2005), and Broadie, Chernov, and Johannes (2007, 2009). Under these parameter values the equity risk premium equals 9.18%, a combination of jump risk premium (3.67%) and diffusion risk premium (5.51%). The risk premium for variance risk equals -3% ($= \bar{v}\zeta$) and the jumps in stock prices of -10% occur with an average intensity of 0.36 on an annual basis.

Our parameter values are close to those mentioned in the recent empirical literature. Santa-Clara and Yan (2010) estimate the ex-ante perceived severity of risks and the associated risk premia. They find that the ex-ante expected jump size is -9.8%, a compensation for stochastic volatility risk of 5%, and a compensation for jump risk of 6.9%. In a closely related study, using a different sample period and estimation method, Pan (2002) identifies a jump premium of 3.5% and

⁵In the sensitivity analysis, we considered a longer horizon of four years.

a volatility premium of 5.5%. Using the newly proposed model free realized and implied volatility measures⁶, Bollerslev, Gibson and Zhou (2011) extract the volatility risk premia parameter (i.e., $\kappa^Q - \kappa$), which turns out to be highly time-varying with an average value of about -1.8. In our baseline parameter setting we take a more conservative $\kappa^Q - \kappa = -1.1$.

Risk Aversion	$\gamma = 5.50$
Habit formation parameter	$h = 0.50$
Riskfree rate	$r = 0.030$
Mean reversion speed	$\kappa = 6.40$
Equity jump intensity	$\lambda = 24.00, \lambda^Q = 48.00$
Equity jump size	$\mu = -0.100$
Volatility of volatility	$\omega = 0.30$
Equilibrium variance	$\bar{v} = 0.0153$
Initial volatility	$\sqrt{V_0} = 0.124$
Compensation equity risk	$\eta = 3.60$
Compensation volatility risk	$\zeta = -2$
Correlation between diffusive equity risk and vairance risk	$\rho = -0.53$

Table: Parameter Values (Base Case)

As mentioned in the aforementioned literature, estimating these risk premia precisely is very difficult, so our choices represent a trade-off between different reported values. However, the results of sensitivity tests shown later in this paper indicate that all our conclusions remain valid under rather broad parameter ranges and even when the volatility risk premium or the jump risk premium is set to zero.

2.5 Solution method

We approximated the distribution of the asset returns using a simulation for an investment horizon of one year which involved 250.000 paths with 1200 time steps per simulation each. We used numerical optimization to find the optimal portfolio in terms of expected utility of final wealth and used exact equations to check local first order optimality conditions.

⁶More recent literature rely on model free approach, extracting information from high-frequency data of the underlying and the model free implied volatility data (e.g. VIX index).

3 Results

3.1 Allocation with only stocks and bonds

Table 2 summarizes our main results. For the base case without derivatives, we find an equity investment corresponding to 44% of the initial wealth and generating a certainty equivalent rate of return (CER) of 5.28%. This portfolio reflects the equity investment of a typical pension fund, which resembles a realistic benchmark for our study. Indeed, we have calibrated the risk aversion parameter γ to achieve this.

The reported values of Value-at-Risk and Tail-Value-at-Risk of -5.22% and -7.79%, the indicated mean, median and standard deviation of the rate of return (7.33%, 7.42% and 6.28% respectively) and the probabilities of earning more than the riskfree rate or even more than the riskfree rate plus inflation (76% and 65% respectively) form the baseline that all subsequent results should be compared to.

3.2 Allocation including equity derivatives

In the second strategy we include an OTM put option in the asset mix with a maturity of 15 months and a strike which equals 95% of the initial equity value. It is interesting to note that instead of direct protection of the stock investment using a long position in the put it is better to reduce stock holdings and sell puts (i.e., a *short* position in OTM puts of -0.79%). This confirms the results reported in a paper by Driessen and Maenhout (2009) that under CRRA utility puts often turn out to be too expensive for the protection that they offer in downward scenarios. The notional of the puts is about 57% of the stocks invested. In this strategy, risk (measured in terms of standard deviation or Value-at-Risk) is reduced by a smaller investment in stocks combined with a shorted put option instead of a reduction achieved by a long position in puts. The reason for this is the negative volatility premium which makes it attractive to have a net short position in derivatives. The overall effect of this combination on the distribution of the funding ratio at the end of the investment horizon is a slight improvement of the Value-at-Risk and a substantial decrease of the standard deviation, with relatively little change in the mean and median of the rate of return. The probabilities of earning the riskfree rate or even earning the riskfree rate plus inflation both increase by a few percentage points. Although the Value-at-Risk improves, the Tail-Value-at-Risk becomes slightly worse due to the payoff needed for the shorted put under scenarios where the stock price decreases substantially. This nicely illustrates the limitations of only looking at the 2.5% quantile as a risk measure without considering the behavior in the tail of the distribution.

In the third strategy we add to the asset mix of stocks and bonds a fixed combination of the OTM put of the previous strategy and a shorted OTM call with the same maturity and a strike at 115% of the initial equity value. If implemented fully, i.e. if one put and one shorted call would be taken for every stock in the investment portfolio, the result would be the *collar* which

has been mentioned before. The rate of return would then be bounded from above and below by predetermined quantities. The combination of both a long and a short position in options may be expected to overcome the relatively high price for the protection offered by put options, by the possibility to sell calls to help the financing of puts. It turns out, however, that in the optimized strategy almost no money is invested in the derivatives. They represent only -0.01% of invested wealth, which corresponds to a notional of 1% of stocks. Therefore their effect on the key statistics for the funding ratio distribution are negligible.

The attractiveness of shorting derivatives rather than taking long positions is also underlined in a strategy where we take the same call and put options as before but this time we allow different investments in put and call and we do not prescribe the position to be long or short in either option. We find a strategy which contains a short call (which corresponds to 60% of stocks) and a short put (which corresponds to 8% of stocks) and roughly equal weights in stocks and bonds. Compared to the performance of the optimal portfolio which contains only puts we see that the probabilities of achieving the riskfree rates with and without inflation increase with 0.3% and 3%, but Value-at-Risk and Tail-Value-at-Risk worsen slightly. The mean is the same but the median improves by 0.7% and the standard deviation is 0.3% lower, indicating that the distribution has not only become more compact but is now also more 'tilting' towards positive values.

3.3 Allocation including variance derivatives

We obtain a more distinct example of this effect in the fourth strategy, where the variance product is added to the asset menu. When shorted it gives direct exposure to variance risk and earns the associated volatility premium⁷. This portfolio is welfare enhancing, with CER up to 5.89% (from 5.28% in the benchmark case), because the investors can make explicit trade-offs between diffusion risk, jump risk and volatility risk and the compensating risk premia. We see that the holding of stock is reduced dramatically (compared to the first strategy which only uses stocks and bonds) as the shorted variance product has a positive correlation with stock. As in the second strategy, the Value-at-risk improves while the Tail-Value-at-Risk deteriorates a bit, but this time we see a spectacular improvement in terms of mean and median of the rate of return (by 0.5% and 1.2% respectively) while the standard deviation is reduced by 0.8% as well. The probabilities of beating the risk-free rate and inflation improve markedly, by 6% and 8% respectively.

If we include both options and the variance product in the asset mix in the sixth and last strategy we find additional small improvement for the certainty equivalent rate of return, the mean, the median and the probability of beating the risk-free rate plus inflation. The optimal strategy contains a similar investment in the variance product but the investment in equity increases from 23.7% to 33.6% and this time we get indeed a construction resembling a collar. A long investment in puts (with 55% stock covered) and a short position in calls (31% of stock covered) together

⁷Under the default parameter setting, the average level of the implied volatility (i.e. the square root of the implied variance product) is 16.6% and the average realized volatility in the stock market is about 13.6% in our model.

reduce the upside potential of the stock returns but protect against adverse scenarios. Since the variance product is part of the asset menu, the overall exposure to volatility risk will be such that volatility risk premia can be earned even though we have a long position in put options. We thus see a combination of volatility as a risk factor that we can invest in while still buying protection for our equity exposure.

3.4 Summary of findings

First, we notice that in an environment where we have stochastic volatility with an associated negative risk premium, direct protection by put options is suboptimal. This is even true if the protection is financed by writing calls on relatively high stock prices. Instead, reducing the risk can be achieved by taking a smaller investment in stocks and *writing* put options. The effect is illustrated in Figure 1 where we show the rate of return of our portfolio as a function of the rate of return of the stock. We see that the exposure to both advantageous and disadvantageous extreme scenarios has indeed been reduced and that more probability mass of the distribution of the wealth will be shifted to the center as a result. This is confirmed by the plot of the distribution of the wealth (Figure 3) which can be found at the end of the paper.

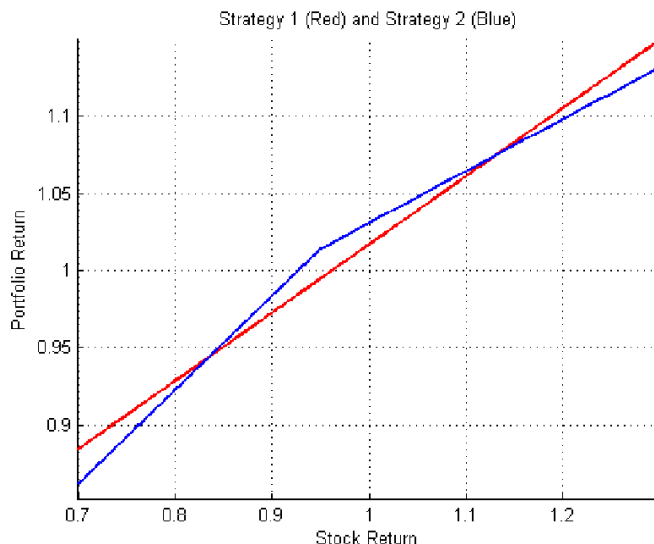


Figure 1: Strategy Returns

The second message is that once derivatives allow us to take a direct exposure to volatility without exposure to the (direction of) stock prices, the investment opportunity set is enlarged and volatility can then be considered to be an asset class on its own. Investors thus can make tradeoffs among distinct risk factors according to their preferences. If the variance product is shorted, it has a positive risk premium (driven by volatility risk and jump risk) and a positive correlation with

stock returns. This is partially based on the direct correlation between volatilities and stock prices but also due to the fact that when stock prices jump downward, the variance product loses value as well; this effect makes the Value-at-risk smaller but the Tail-Value-at-risk a bit higher compared to the situation where no variance products have been added to the asset mix. However, this can be remedied as we will show in the next section when we discuss robustness.

4 Robustness checks

4.1 Controlling tail risk

When we add a constraint that tail risk for our optimized portfolio should not exceed the value found for the optimal portfolio which contains only stocks and bonds, we find that higher income security and welfare enhancement are still obtainable using our extended asset menu. Table 3 shows an example, where we take the best portfolio of the preceding section but reduce the equity holdings by 5% and increase bond holdings by the same amount. The resulting portfolio dominates the optimal stock and bond portfolio along all evaluation criteria including the two tail risk indicators (TVaR and VaR). Of course this portfolio is (slightly) suboptimal in terms of certainty equivalent rate of return, which emphasizes that the optimization programme in terms of expected utility of terminal wealth is a useful tool to generate candidate portfolios but that it may be beneficial to tweak the results in order to improve along different performance criteria which are more important in practice.

Since derivatives are marked-to-market throughout the holding period, the daily movements in the profit and loss (P&L) accounts of the derivatives might be a concern for investors. To address this issue we investigated the tail risks in the P&L of the variance swap during the holding period in our model. In our simulations, the distribution of the P&L turns out to show rather limited downside risk, which can be explained by the relatively long maturity of the variance product.

4.2 Sensitivity analysis

Although our discussion has been based on results for the specific parameter setting that we chose as our base case, these results seem to be rather robust in the face of parameter changes. Table 4 to 9 in the Appendix show that although the precise asset holdings for the optimal portfolios will differ under such changes, the structure of the optimal solution, and therefore our conclusions, remain valid.

Table 4 shows the optimal portfolio with an initially high volatility environment (with the initial volatility doubled). The allocation to stocks and bonds are not affected, but the allocation to derivatives are increased. This is due to the mean reversion feature of the assumed stochastic volatility model.

Table 5, 6 and 7 show the results with reduced size of premia for volatility risk, jump risk and diffusive risk respectively. The structure of the optimal solution remains the same, and the improvements to the evaluation criteria are still sizeable.

Table 8 and 9 push the sensitivity analysis to an extreme by assuming zero risk premium for volatility risk or jump risk. In such environment, the qualitative messages still remain although quantitatively reduced in sizes. This demonstrates that welfare enhancement critically depends on the volatility and jump risk premia believes of the investors.

4.3 Longer horizon and rolling over

Since pension funds have a long term investment horizon, we extended the horizon to four years and investigated two cases.

In the first case, the investment horizon as well as the time-to-maturity of the derivatives are both extended, so that the investor can follow the same buy-and-hold strategy over the longer horizon as in the baseline case. Table 10 shows that the portfolio which includes variance derivatives improves the performance along all criteria, including the tail risk indicators, over a four-year period. The optimal portfolio has a relatively large shorted position in variance product.

Since equity and variance derivatives with longer maturities are less liquid, we also investigate a second case which involves rolling over the one-year-to-maturity variance product and the options on a yearly bases. Table 11 shows some promising results for this 'rolling over strategy'. The position in variance product is modest, but the improvements along almost all criteria remain and the structure of the optimal portfolio is similar to the one for the baseline case.

5 Practical Implications

The strategies discussed in the previous section yield superior results in terms of the performance measures. It is therefore important to consider to what extent the reported efficiency gains can also be captured when implemented for real life pension plans. By and large, potential implementation issues may arise due to constraints in terms of market liquidity and potential mismatches between existing portfolio structure and the structure of the proposed derivatives program.

Although most of the derivative products suggested in section 3 are frequently traded Over The Counter (OTC) as well as on exchanged-traded platforms, some of the products may be substantially less liquidly available than their underlying equity portfolio. As an example, variance products are traded in OTC markets, but for larger pension plans (say, plans in excess of USD 10bn Assets under Management) liquidity may not yet be sufficient to implement strategies for meaningful notionals, even under normal market conditions. Plain-vanilla equity derivatives (single or basket index puts and calls) are more liquidly available, although for large investors, transaction programs may take

a substantial amount of time before they are completed, unless one is willing to accept significant implementation costs - yielding lower efficiency gains. Once derivative products become part of the overall strategy, investors should also manage liquidity risk in trading derivatives, i.e. the risk that the intended strategy can not be maintained in the future when the derivative contracts expire. A potential mitigating strategy may be to diversify strategies in terms of their roll over horizon, such that not the entire notional of the strategy needs to be rolled forward at a single moment in time.

As a second class of potential implementation issues, it may be cumbersome to align the derivative strategy with other parts of the existing portfolio. As an example, basis risk may arise due to equity investments which are actively managed, whereas the underlying value of the equity derivative relates to a market benchmark (such as the S&P 500 index). For internationally diversified equity portfolios, the benchmark may not be available in a single derivative contract, unless one is willing to trade OTC basket options tailored towards the investor's specific equity portfolio. In addition, derivatives on internationally diversified portfolios may require different currency hedging than the underlying portfolio, which may be subject to intermediate portfolio rebalancing.

There may also be differences in treating call and put options. In the presence of active management, long positions in put options can be combined with an existing actively managed portfolio, if one is willing to accept the basis risk. Solutions, however, which require less investment in the underlying portfolio, replaced with a position in long call options, can not be combined with active management at all. Hence, some of the strategies may trigger transition or opportunity costs which need to be taken into account when evaluating derivatives strategies as well.

6 Conclusion

In this paper we have investigated the effect of including derivatives in the asset menu for a portfolio optimization problem faced by a pension fund. We find that when volatility risk premia and the possibility of sudden downward jumps in stock prices are taken account of, standard option strategies which use puts to protect the portfolio may be suboptimal under the volatility and jump risk premia that are reported in the literature. However, by writing instead of buying put options or by exploiting the possibility to invest in the floating leg of variance swaps, marked improvements can be achieved in certainty equivalent rates of return and among other performance measures that pension fund managers are more familiar with. Sensitivity studies indicate that our conclusions remain valid under rather broad parameter ranges and even when the volatility risk premium or the jump risk premium is set to zero. A 'rolling over strategy' for the derivatives which was implemented as part of our robustness checks, shows further promising results.

More work remains to be done to facilitate the implementation of ALM strategies which focus on allocations in terms of risk premia instead of particular assets. However, we believe that such a focus will lead to better understanding and hence better control over the exposure to different

risks. Volatility-based derivatives are a good illustration of the possibilities in this direction, but we strongly believe that the more general principle of explicitly separating exposures to different risk drivers transcends that particular example.

7 References

D.S. Bates. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options. *Review of Financial Studies*, 9(1):69–107, 1996.

G. Bekaert, E. Engstrom, Y. Xing. Risk, uncertainty and asset prices. *Journal of Financial Economics* 91:39–82, 2009.

D. Blake, A. Cairns, and K. Dowd. Turning pension plans into pension planes: What investment strategy designers of defined contribution pension plans can learn from commercial aircraft designers. City University Pensions Institute discussion papers, (PI-806), 2008.

T. Bollerslev, M. Gibson, and H. Zhou. Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities, *Journal of Econometrics*, 160: 235–245, 2011.

M. Broadie, M. Chernov, and M. Johannes. Model specification and risk premia: Evidence from futures options. *Journal of Finance*, 62(3):1453–1490, 2007.

M. Broadie, M. Chernov, and M. Johannes. Understanding index option returns. *Review of Financial Studies*, 22(11):4493–4529, 2009.

M. Broadie and O. Jain, A. The effect of jumps and discrete sampling on volatility and variance swaps. *International Journal of Theoretical*

P. Carr, L. Wu. Variance risk premiums. *Review of Financial Studies* 22, 1311-1341, 2009.

J. Driessen, P. Maenhout, and G. Vilkov. The price of correlation risk: Evidence from equity options. *The Journal of Finance*, 64(3):1377–1406, 2009.

J.-C. Duan and Y. C.Y. Price and volatility dynamics implied by the vix term structure. Working paper, National University of Singapore, 2011.

B. Eraker, M. Johannes, and N. Polson. The impact of jumps in volatility and returns. *The Journal of Finance*, 58(3):1269–1300, 2003.

J.A. Faias and P. Santa-Clara. Optimal option portfolio strategies. Working paper, 2011.

S. Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6:327–343, 1993.

J. Liu and J. Pan. Dynamic derivative strategies. *Journal of Financial Economics*, 69:401–430, 2003.

R.C. Merton. Lifetime portfolio selection under uncertainty: the continuous-time case. *Rev. Econom. statist*, 247–257, 1969.

R.C. Merton. Solutions for the future. *Pensions Insight*, 2010.

J. Pan. The jump-risk premia implicit in options: evidence from an integrated time-series study. *Journal of Financial Economics*, 63(1):3 – 50, 2002.

N. Schurhoff and A. Ziegler. Variance risk, financial intermediation, and the cross-section of expected option returns. Technical Report, 2011.

V. Todorov. Variance Risk-Premium Dynamics: The Role of Jumps, *Review of Financial Studies* 23 (1): 345-383, 2010.

8 Appendix

8.1 Valuation of derivative instruments

8.1.1 Stochastic Kernel and Characteristic Function

To price vanilla equity options and variance swaps in our economy we need to determine the asset price distributions under a martingale measure \mathbb{Q} . The pricing kernel process π is constructed using $\pi_0 = 1$ and

$$\frac{d\pi_t}{d\pi_{t-}} = -r dt - \sqrt{V_t}(\eta dY_t + \zeta dZ_t) + \left(\frac{\lambda^Q}{\lambda} - 1\right) (dN_t - \lambda V_t dt)$$

which shows that the measure \mathbb{Q} defined by $\frac{d\mathbb{Q}}{d\mathbb{P}} = e^{rT} \pi_T$ is a martingale measure for this market since the Ito rule for discontinuous semi-martingales gives that

$$d(\pi_t S_t) / (\pi_{t-} S_{t-}) = \sqrt{V_t}(-\zeta dZ_t + (1 - \eta)dY_t) + ((1 + \mu)\frac{\lambda^Q}{\lambda} - 1)(dN_t - \lambda V_t dt).$$

Writing $\tilde{\rho} = \sqrt{1 - \rho^2}$ and defining the two \mathbb{Q} -Brownian motions $Y_t^Q = Y_t + \eta \int_0^t \sqrt{V_u} du$ and $Z_t^Q = Z_t + \zeta \int_0^t \sqrt{V_u} du$ we find for the logarithmic price process $X_t = \ln S_t$ under \mathbb{Q}

$$\begin{aligned} dX_t &= (r - \frac{1}{2}V_t - \mu\lambda^Q V_t)dt + \sqrt{V_t}dY_t^Q + (\ln(1 + \mu))dN_t \\ dV_t &= \kappa^*(\bar{v}^* - V_t)dt + \omega\sqrt{V_t}(\rho dY_t^Q + \tilde{\rho}dZ_t^Q) \end{aligned}$$

if we take $\kappa^* = \kappa + \omega(\rho\eta + \tilde{\rho}\zeta)$ and $\bar{v}^* = \bar{v}\kappa/\kappa^*$. We define the characteristic function $\phi(x, v, t) = \mathbb{E}^{\mathbb{Q}}[e^{uX_T} | X_t = x, V_t = v]$ for all $u \in \mathbb{C}$ and since $\phi(X_t, V_t, t)$ must be a martingale under \mathbb{Q} we conclude from

$$\begin{aligned}
d\phi(X_t, V_t, t) &= [(r - \frac{1}{2}V_t - \mu\lambda^Q V_t) \frac{\partial\phi}{\partial x}(X_t, V_t, t) + \kappa^*(\bar{v}^* - V_t) \frac{\partial\phi}{\partial v}(X_t, V_t, t)]dt \\
&+ [\frac{1}{2}V_t \frac{\partial^2\phi}{\partial x^2}(X_t, V_t, t) + \rho\omega V_t \frac{\partial^2\phi}{\partial x\partial v}(X_t, V_t, t) + \frac{1}{2}\omega^2 V_t \frac{\partial^2\phi}{\partial v^2}(X_t, V_t, t)]dt \\
&+ \frac{\partial\phi}{\partial t}(X_t, V_t, t)dt + \frac{\partial\phi}{\partial x}(X_t, V_t, t)\sqrt{V_t}dY_t^Q + \frac{\partial\phi}{\partial v}(X_t, V_t, t)\sqrt{V_t}\omega(\rho dY_t^Q + \tilde{\rho}dZ_t^Q) \\
&+ [\phi(X_t + \ln(1 + \mu), V_t, t) - \phi(X_t, V_t, t)]dN_t
\end{aligned}$$

that we must have

$$(r - \frac{1}{2}v - \mu\lambda^Q v)\phi_x + \kappa^*(\bar{v}^* - v)\phi_v + v(\frac{1}{2}\phi_{xx} + \rho\omega\phi_{xv} + \frac{1}{2}\omega^2\phi_{vv}) + \phi_t = -\lambda^Q v\phi\mathbb{E}^Q(e^{u\ln(1+\mu)} - 1).$$

We look for a solution of the form $\phi(x, v, t) = \exp[ux + A(u, T-t) + vB(u, T-t)]$ and substitution gives, after division by ϕ

$$\begin{aligned}
&(r - \frac{1}{2}v - \mu\lambda^Q v)u + v(\frac{1}{2}u^2 + \rho\omega uB(u, T-t) + \frac{1}{2}\omega^2 B^2(u, T-t)) \\
&+ \kappa^*(\bar{v}^* - v)B(u, T-t) - v\frac{\partial B}{\partial t}(u, T-t) - \frac{\partial A}{\partial t}(u, T-t) \\
&= -\lambda^Q v[(e^{u\ln(1+\mu)} - 1)]
\end{aligned}$$

This means we must have that $A(u, 0) = B(u, 0) = 0$ and

$$\begin{aligned}
\frac{\partial B}{\partial t}(u, \tau) &= a + bB(u, \tau) + cB^2(u, \tau) \\
\frac{\partial A}{\partial t}(u, \tau) &= ur + \kappa^*\bar{v}^*B(\tau)
\end{aligned}$$

with

$$\begin{aligned}
a &= (-\frac{1}{2} - \mu\lambda^Q)v + \frac{1}{2}u^2 + \lambda^Q v[e^{u\ln(1+\mu)} - 1] \\
b &= \rho\omega u - \kappa^* \\
c &= \frac{1}{2}\omega^2
\end{aligned}$$

The solution of these ordinary differential equations of Ricatti type are

$$B(u, \tau) = \frac{2a(1 - e^{-D\tau})}{2D - (D + b)(1 - e^{-D\tau})}$$

$$A(u, \tau) = ur\tau + \kappa^* \bar{v}^* \cdot \frac{-1}{2c} \left[(D + b)\tau + 2 \ln\left(1 - \frac{D + b}{2D} (1 - e^{-D\tau})\right) \right]$$

if we define $D = \sqrt{b^2 - 4ac}$.

8.1.2 European Vanilla options

By the Lévy inversion formula we find for the price of a call option with strike K that $C = S_t P(1) - K e^{-r(T-t)} P(0)$ if we choose

$$P(1) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{du}{u} \operatorname{Im}(e^{iu[\ln(K/S_t) - r(T-t)] + A(1-iu, T-t) - (1-iu)r(T-t) + V_t B(1-iu, T-t)}),$$

$$P(0) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{du}{u} \operatorname{Im}(e^{iu[\ln(K/S_t) - r(T-t)] + A(0-iu, T-t) - (0-iu)r(T-t) + V_t B(0-iu, T-t)}).$$

8.1.3 Variance Swap

The payoff of a variance swap monitored at times $T_0 = t_0 < t_1 < \dots < t_n = T_e$ equals

$$\frac{1}{T_e - T_0} \sum_{i=0}^{n-1} \left(\ln \frac{S_{t_{i+1}}}{S_{t_i}} \right)^2 - K$$

where K , the variance swap strike is usually chosen in such a way that its price at time T_0 equals zeros. If we take $t_i = T_0 + (T_e - T_0)/n$, this converges to $(\langle \ln S \rangle_{T_e} - \langle \ln S \rangle_{T_0}) / (T_e - T_0) - K$ for $n \rightarrow \infty$. So for our asset dynamics we find for a continuously monitored variance swap

$$\begin{aligned} K &= \mathbb{E}^{\mathbb{Q}} \left[\frac{\langle \ln S \rangle_{T_e} - \langle \ln S \rangle_{T_0}}{T_e - T_0} \mid \mathcal{F}_{T_0} \right] \\ &= \mathbb{E}^{\mathbb{Q}} \left[\int_{T_0}^{T_e} V_u du + \mid \mathcal{F}_{T_0} \right] + \mathbb{E}^{\mathbb{Q}} \left[\sum_{i=N_{T_0}}^{N_{T_e}} \ln(1 + \mu)^2 \mid \mathcal{F}_{T_0} \right] \\ &= \frac{1}{T_e - T_0} (1 + \lambda^Q (\ln(1 + \mu))^2) \int_{T_0}^{T_e} \mathbb{E}^{\mathbb{Q}} [V_u \mid \mathcal{F}_{T_0}] du \\ &= (1 + \lambda^Q (\ln(1 + \mu))^2) \left(\bar{v}^* + \frac{V_{T_0} - \bar{v}^*}{\kappa^* (T_e - T_0)} (1 - e^{-\kappa^* (T_e - T_0)}) \right) \end{aligned}$$

where we have used conditioning on the path $\{V_u, T_0 \leq u \leq T_e\}$ in the third inequality and a standard result on the integrated variance in CIR models in the last equality (see, for example,

Broadie and Jain (2008)).

Define $L = 1 + \lambda^Q((\ln(1 + \mu))^2)$, $D_t = e^{-r(T_e - t)}/(T_e - T_0)$ and $M_t = e^{-\kappa^*(T_e - t)}$. A product with payoff $H_{T_e} = (\langle \ln S \rangle_{T_e} - \langle \ln S \rangle_{T_0})/(T_e - T_0)$ at a time $T_e > T_0$ has thus a value at time $t \in [T_0, T_e]$ equal to

$$\begin{aligned} H_t &= e^{-r(T_e - t)} \mathbb{E}^Q[H_{T_e} | \mathcal{F}_t] = D_t(\langle \ln S \rangle_t - \langle \ln S \rangle_{T_0}) + L \int_t^{T_e} \mathbb{E}^Q[V_u | \mathcal{F}_t] du \\ &= D_t(\langle \ln S \rangle_t - \langle \ln S \rangle_{T_0}) + L \left[(T_e - t)\bar{v}^* + \frac{V_t - \bar{v}^*}{\kappa^*} (1 - e^{-\kappa^*(T_e - t)}) \right]. \end{aligned}$$

8.2 Figures

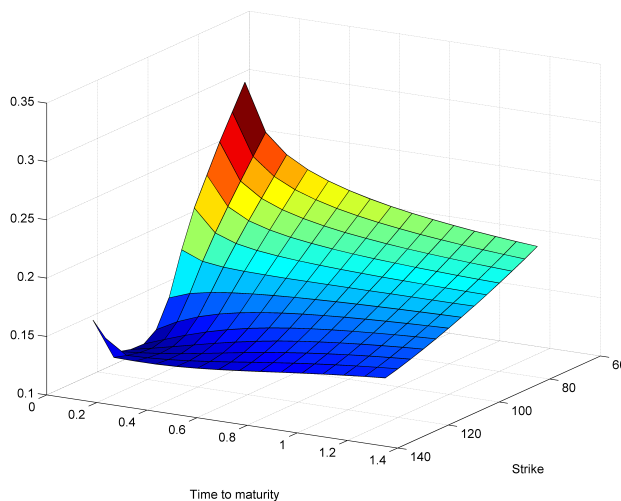


Figure 2: Implied Volatility (Base Case)

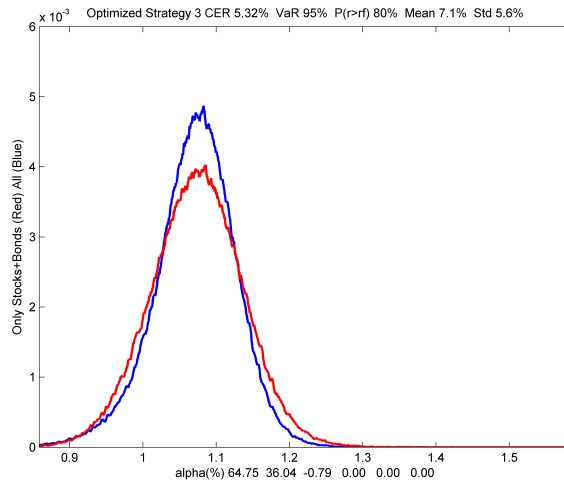


Figure 3: Portfolio return distribution with OTM put

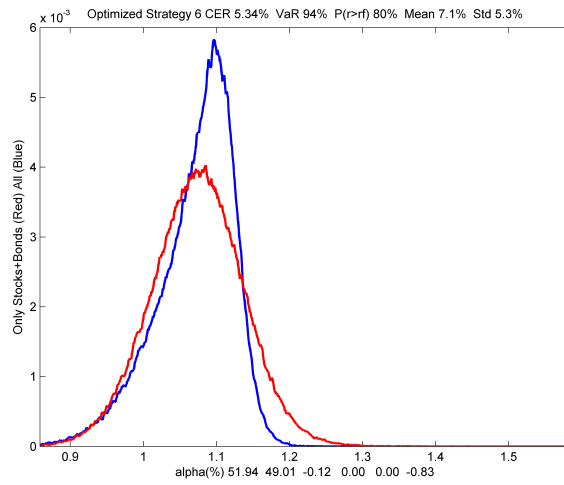


Figure 4: Portfolio return distribution with OTM put
and OTM call

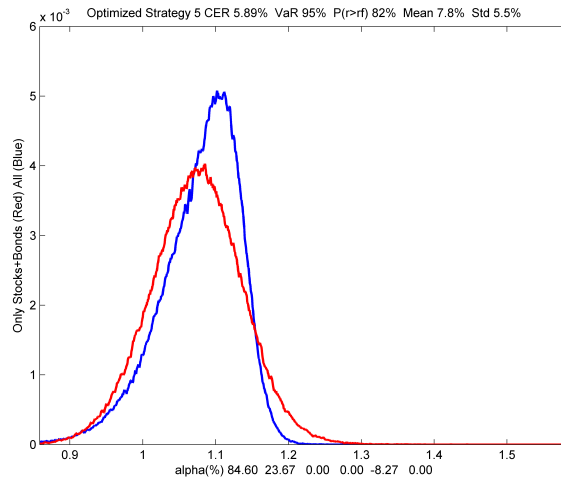


Figure 5: Portfolio return distribution with ATM Variance product

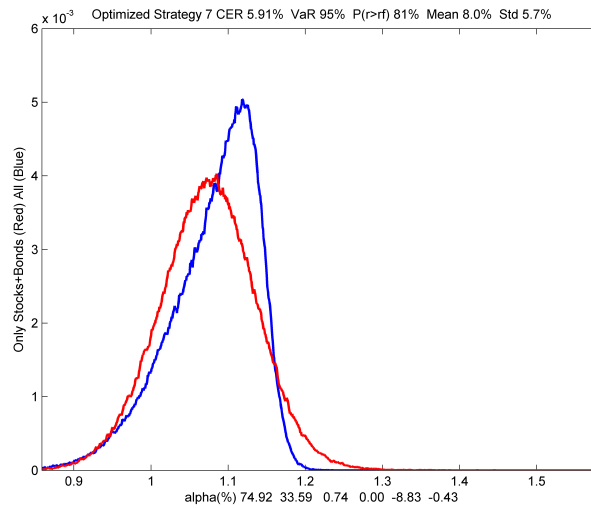


Figure 6: Portfolio return distribution with OTM put, OTM call, and variance product

8.3 Tables

In the tables we report results for the different cases. Results are provided for optimized strategies

1. Bonds and Stocks
2. Bonds, Stocks and out-of-the-money puts
3. Bonds, Stocks and a fixed combination of a long position in out-of-the-money puts and a short position in out-of-the-money calls
4. Bonds, Stocks and a the variance product (payoff equals the average realized variance during the investment period)
5. Bonds, Stocks, out-of-the-money puts and out-of-the-money calls
6. Bonds, Stocks, out-of-the-money puts, the variance product and out-of-the-money calls

α_B	Percentage invested in bonds
α_S	Percentage invested in stocks
α_P	Percentage invested in out-of-the-money puts
α_{C_o}	Percentage invested in combination of out-of-the-money puts and (shorted) out-of-the-money calls
α_V	Percentage invested in variance product
α_{C_a}	Percentage invested in out-of-the-money calls
CER	Certainty equivalent rate of return
VaR	Value-at-Risk at 2.5%
TVaR	Tail-Value-at-Risk at 2.5%
μ_R	Mean rate of return
m_R	Median rate of return
σ_R	Standard deviation rate of return
P_r	Probability of earning more than the riskfree rate
P_i	Probability of earning more than the riskfree rate plus the inflation rate

Table 2: Maturity $T = 1$, baseline results

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
55.91 %	44.09 %					5.28 %	-5.22 %	-7.79 %	7.33 %	7.42 %	6.28 %	76.08 %	65.22 %
64.75 %	36.04 %	-0.79 %				5.32 %	-5.06 %	-8.36 %	7.12 %	7.40 %	5.56 %	79.53 %	67.78 %
56.83 %	43.18 %		-0.01 %			5.28 %	-5.20 %	-7.80 %	7.33 %	7.38 %	6.28 %	76.09 %	65.09 %
84.60 %	23.67 %			-8.27 %		5.89 %	-4.96 %	-8.37 %	7.81 %	8.69 %	5.48 %	82.09 %	73.11 %
51.94 %	49.01 %	-0.12 %			-0.83 %	5.34 %	-5.50 %	-8.54 %	7.10 %	8.11 %	5.30 %	79.86 %	70.72 %
74.92 %	33.59 %	0.74 %			-0.43 %	5.91 %	-5.45 %	-8.64 %	7.99 %	9.09 %	5.74 %	81.21 %	73.16 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.0153, \bar{v}^* = 0.0184 \sqrt{V_0} = 0.124$

Table 3: Maturity $T = 1$ Adjusted Portfolio α_s 5% lower

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
55.91 %	44.09 %					5.28 %	-5.22 %	-7.79 %	7.33 %	7.42 %	6.28 %	76.08 %	65.22 %
79.92 %	28.59 %	0.74 %			-0.43 %	5.87 %	-4.66 %	-7.63 %	7.51 %	8.59 %	5.13 %	81.53 %	72.57 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.0153, \bar{v}^* = 0.0184 \sqrt{V_0} = 0.124$

Table 4: Maturity $T = 1$, Initial volatility doubled

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
55.73 %	44.27 %					6.38 %	-5.83 %	-8.65 %	9.50 %	9.41 %	7.95 %	79.48 %	71.44 %
63.96 %	36.93 %	-0.90 %				6.43 %	-5.54 %	-9.08 %	9.14 %	9.27 %	7.01 %	82.50 %	74.07 %
56.25 %	43.76 %		-			6.38 %	-5.81 %	-8.65 %	9.50 %	9.39 %	7.96 %	79.48 %	71.38 %
87.20 %	24.37 %			-11.57 %		7.29 %	-5.06 %	-8.82 %	10.13 %	10.86 %	6.79 %	85.41 %	78.97 %
52.96 %	48.08 %	-0.19 %			-0.85 %	6.44 %	-5.96 %	-9.27 %	9.08 %	9.91 %	6.64 %	82.68 %	75.83 %
78.05 %	33.80 %	0.90 %		-12.27 %	-0.48 %	7.31 %	-5.56 %	-9.10 %	10.38 %	11.33 %	7.10 %	84.51 %	78.52 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 48.00,$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.018, \sqrt{V_0} = 0.247$

Table 5: Maturity $T = 1$, Volatility Risk Premium halved

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
55.91 %	44.09 %					5.28 %	-5.22 %	-7.79 %	7.33 %	7.42 %	6.28 %	76.08 %	65.22 %
62.48 %	38.10 %	-0.58 %				5.30 %	-5.13 %	-8.25 %	7.16 %	7.39 %	5.74 %	78.54 %	66.98 %
57.85 %	42.17 %		-0.02 %			5.28 %	-5.17 %	-7.82 %	7.32 %	7.34 %	6.29 %	76.12 %	64.94 %
76.74 %	29.27 %			-6.02 %		5.60 %	-5.14 %	-8.33 %	7.51 %	8.16 %	5.64 %	80.12 %	70.47 %
54.31 %	46.36 %	-0.16 %			-0.51 %	5.31 %	-5.41 %	-8.36 %	7.13 %	7.84 %	5.54 %	78.88 %	69.01 %
71.32 %	34.79 %	0.42 %		-6.30 %	-0.22 %	5.60 %	-5.38 %	-8.38 %	7.59 %	8.38 %	5.77 %	79.55 %	70.53 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.57, \lambda = 24.00, \lambda^Q = 48.00,$

$\mu = -0.100; \eta = 3.60, \zeta = -1.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.018, \sqrt{V_0} = 0.124$

Table 6: Maturity $T = 1$, Jump Risk Premium halved

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
64.41 %	35.59 %					4.45 %	-4.36 %	-6.41 %	5.78 %	5.88 %	5.03 %	71.73 %	57.05 %
71.58 %	29.05 %	-0.63 %				4.48 %	-4.54 %	-7.26 %	5.68 %	5.97 %	4.55 %	75.49 %	59.25 %
66.93 %	33.10 %		-0.03 %			4.45 %	-4.34 %	-6.50 %	5.78 %	5.78 %	5.04 %	71.82 %	56.42 %
83.11 %	22.41 %			-5.52 %		4.71 %	-4.68 %	-7.46 %	6.03 %	6.67 %	4.69 %	76.47 %	63.60 %
63.60 %	37.12 %	-0.23 %			-0.48 %	4.49 %	-4.75 %	-7.28 %	5.67 %	6.42 %	4.43 %	75.72 %	62.22 %
79.16 %	26.37 %	0.21 %		-5.53 %	-0.21 %	4.71 %	-4.78 %	-7.47 %	6.04 %	6.81 %	4.68 %	76.42 %	64.20 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 36.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.018, \sqrt{V_0} = 0.124$

Table 7: Maturity $T = 1$, Diffusive Risk Premium halved

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
68.74 %	31.26 %					4.12 %	-3.76 %	-5.57 %	5.14 %	5.23 %	4.41 %	69.45 %	52.10 %
79.76 %	21.25 %	-1.00 %				4.18 %	-4.19 %	-7.02 %	5.06 %	5.50 %	3.77 %	76.76 %	56.29 %
71.97 %	28.07 %		-0.04 %			4.12 %	-3.76 %	-5.71 %	5.13 %	5.12 %	4.42 %	69.58 %	51.13 %
101.96 %	8.00 %			-9.96 %		4.95 %	-4.61 %	-7.78 %	6.24 %	7.36 %	4.40 %	79.48 %	67.40 %
67.19 %	33.94 %	-0.36 %			-0.77 %	4.20 %	-4.48 %	-7.02 %	5.09 %	6.15 %	3.73 %	76.33 %	61.75 %
95.13 %	14.94 %	0.52 %		-10.31 %	-0.28 %	4.95 %	-4.88 %	-7.90 %	6.33 %	7.49 %	4.59 %	78.63 %	67.40 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.61, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 1.80, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.017, \sqrt{V_0} = 0.124$

Table 8: Maturity $T = 1$, Jump Risk Premium Zero

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
73.13 %	26.87 %					3.83 %	-3.07 %	-4.63 %	4.58 %	4.67 %	3.78 %	67.08 %	46.39 %
77.38 %	22.98 %	-0.36 %				3.84 %	-3.40 %	-5.40 %	4.54 %	4.78 %	3.56 %	70.32 %	47.31 %
74.94 %	25.09 %		-0.02 %			3.83 %	-3.09 %	-4.73 %	4.58 %	4.61 %	3.78 %	67.19 %	45.67 %
79.42 %	22.45 %			-1.87 %		3.85 %	-3.29 %	-5.12 %	4.60 %	4.85 %	3.67 %	69.11 %	48.33 %
73.25 %	27.15 %	-0.17 %			-0.23 %	3.84 %	-3.46 %	-5.35 %	4.54 %	4.99 %	3.51 %	70.47 %	49.82 %
78.18 %	23.69 %	-0.06 %			-0.15 %	3.86 %	-3.45 %	-5.38 %	4.57 %	5.01 %	3.55 %	70.54 %	50.16 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 24.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.018, \sqrt{V_0} = 0.124$

Table 9: Maturity $T = 1$, Vol Risk Premium Zero

α_B	α_S	α_P	α_{Co}	α_V	α_{Ca}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
55.91 %	44.09 %					5.28 %	-5.22 %	-7.79 %	7.33 %	7.42 %	6.28 %	76.08 %	65.22 %
60.13 %	40.24 %	-0.36 %				5.29 %	-5.18 %	-8.11 %	7.21 %	7.40 %	5.93 %	77.58 %	66.27 %
57.78 %	42.24 %		-0.02 %			5.28 %	-5.17 %	-7.81 %	7.33 %	7.34 %	6.29 %	76.12 %	64.94 %
69.02 %	34.76 %			-3.78 %		5.40 %	-5.23 %	-8.17 %	7.34 %	7.76 %	5.83 %	78.37 %	68.13 %
55.61 %	44.81 %	-0.14 %			-0.28 %	5.29 %	-5.33 %	-8.16 %	7.19 %	7.64 %	5.80 %	77.84 %	67.41 %
66.56 %	37.25 %	0.19 %			-0.09 %	5.40 %	-5.31 %	-8.13 %	7.37 %	7.86 %	5.89 %	78.07 %	68.18 %

$\gamma = 5.50, h = 0.50, T = 1.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.83, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 3.60, \zeta = 0.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.017, \sqrt{V_0} = 0.124$

Table 10: Maturity $T = 4$

α_B	α_S	α_P	α_{C_0}	α_V	α_{C_a}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
51.22 %	48.78 %				24.23 %	2.50 %	-2.20 %	36.62 %	34.92 %	19.94 %	90.57 %	79.39 %	
58.45 %	42.75 %	-1.20 %			24.39 %	4.09 %	-1.56 %	34.95 %	33.53 %	17.63 %	92.24 %	80.30 %	
39.21 %	61.95 %		-1.16 %		24.25 %	2.03 %	-2.65 %	35.94 %	34.63 %	18.70 %	91.11 %	80.94 %	
103.91 %	27.97 %			-31.88 %	27.62 %	6.82 %	0.79 %	37.43 %	37.43 %	15.57 %	94.88 %	87.03 %	
49.05 %	53.01 %	-0.39 %			24.40 %	3.66 %	-1.98 %	34.57 %	33.44 %	16.83 %	92.44 %	81.67 %	
98.70 %	34.93 %	1.18 %			27.72 %	5.86 %	-0.06 %	38.74 %	38.75 %	16.78 %	94.36 %	86.80 %	

$\gamma = 5.50, h = 0.50, T = 4.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.015, \bar{v}^* = 0.018, \sqrt{V_0} = 0.124$

Table 11: Maturity $T = 4$ Roll Over

α_B	α_S	α_P	α_{C_0}	α_V	α_{C_a}	CER	VaR	TVaR	μ_R	m_R	σ_R	P_r	P_i
52.21 %	47.79 %				24.13 %	2.75 %	-2.08 %	36.15 %	34.49 %	19.51 %	90.64 %	79.34 %	
53.35 %	46.80 %	-0.15 %			24.15 %	3.07 %	-1.89 %	35.83 %	34.21 %	19.13 %	90.85 %	79.39 %	
52.20 %	47.80 %				24.13 %	2.75 %	-2.08 %	36.15 %	34.49 %	19.51 %	90.64 %	79.35 %	
61.77 %	40.99 %		-2.76 %		25.20 %	5.25 %	0.24 %	35.14 %	33.88 %	17.12 %	92.67 %	81.36 %	
53.29 %	46.88 %	-0.15 %		-0.01 %	24.15 %	3.08 %	-1.89 %	35.81 %	34.22 %	19.07 %	90.86 %	79.42 %	
60.78 %	41.87 %	0.12 %			25.22 %	4.99 %	0.06 %	35.43 %	34.14 %	17.44 %	92.50 %	81.34 %	

$\gamma = 5.50, h = 0.50, T = 4.00, r = 0.030, \kappa = 6.40, \kappa^* = 5.32, \lambda = 24.00, \lambda^Q = 48.00$

$\mu = -0.100; \eta = 3.60, \zeta = -2.00, \omega = 0.30, \bar{v} = 0.0153, \bar{v}^* = 0.0184, \sqrt{V_0} = 0.124$