



International Centre for
Pension Management



Rotman School of Management
UNIVERSITY OF TORONTO

ICPM Sponsored Research

An Internal Swap Market between Equity Returns and Wage Growth: Can it Enhance the Welfare of Defined Contribution Plan Participants?

Jiajia Cui¹ and Eduard Ponds²

April 30, 2011

The authors acknowledge ICPM for the research grant. The authors are very grateful for the helpful comments by David Blake, Dirk Broeders, Barbara Zvan, Malcolm Hamilton, and seminar participants at the Rotman ICPM Discussion Forum in Toronto, June 9-10, 2010

¹ University of Amsterdam, Netspar and APG, email: J.Cui@uva.nl

² University of Tilburg, Netspar, and APG, email: Eduard.Ponds@apg.nl

For Further Information:

Rotman International Centre for Pension Management

151 Bloor Street West, Suite 702
Toronto ON M5S 1S4 CANADA

Tel: 416.925.4153

Fax: 416.925.7377

icpm@rotman.utoronto.ca

www.rotman.utoronto.ca/icpm

An Internal Swap Market between Equity Returns and Wage Growth: Can it Enhance the Welfare of Defined Contribution Plan Participants?¹

Jiajia Cui² and Eduard Ponds³

April 30, 2011

Abstract

There is a worldwide trend to substitute defined contribution pension plans for defined benefit plans, which are typically indexed to wage or price inflation. A problem with this development is that participants in defined contribution plans have limited access or no access at all to inflation-linked assets, especially wage-linked claims. This article demonstrates that the creation of an internal market that swaps equity-related returns for wage-linked income streams helps to overcome the market incompleteness regarding wage-linked securities. The result is an improvement in the welfare of both younger and older defined contribution plan members.

Keywords

Defined Contribution Plan, Internal Swap Market, Life-cycle Theory, Pension Fund, Wage-Return Swap, Wage-Linked Exposure

¹ The authors acknowledge ICPM for the research grant. The authors are very grateful for the helpful comments by David Blake, Dirk Broeders, Barbara Zvan, Malcolm Hamilton, and seminar participants at the Rotman ICPM Discussion Forum in Toronto, June 9-10, 2010

² University of Amsterdam, Netspar and APG, email: J.cui@uva.nl

³ University of Tilburg, Netspar, and APG, email: eduard.ponds@apg.nl

Implications of the Move to Defined Contribution Plans

The importance of individual defined contribution (DC) pension plans worldwide has grown substantially. This has come at the expense of the relative significance of social security and private sector funded defined benefit (DB) plans. DB plan benefits in retirement are based on the number of years of contribution and a reference wage, which is often related to individual's own past wages. Moreover, during retirement, benefits typically may be indexed for a reference variable, for example wage growth or price inflation. Whereas DB plans are tightly linked to wages, pensions from DC plans on the other hand depend on capital market returns. The sum of contributions plus investment proceeds on these contributions determines pension wealth at retirement. This pension wealth ideally should be converted into annuities (Yaari 1965; Davidoff et al. 2005).

Viewed from the perspective of lifecycle investment theory (Bodie et al., 2007), there are a number of disadvantages of this move away from wage-related pension benefits in favor of return-related benefits:

- DB plans have different risk-return characteristics as DC plans, so a portfolio of retirement income provisions consisting of both DB and DC plans may be welfare-improving as retirement income does not rely on either wage growth or capital market performance but on a combination (Matsen and Thogerson, 2004).
- Wage-indexed DB plans overcome the serious market incompleteness regarding the lack of supply of wage-indexed or price-indexed bonds.⁴
- Wage-indexed DB plans organize intergenerational risk sharing as the elderly with their funded DB claims share in wage growth risk. In funded schemes with well-organized intergenerational risk sharing, the young and the old may benefit from more risk taking in capital market because of the extension of the investment horizon (Cui et al., 2011; Gollier 2008).

⁴ The valuation of wage-linked claims in incomplete market setting is discussed in De Jong (2008).

We propose the creation of an internal market of wage-linked swaps, where the aggregate growth rates of wage can be exchanged for risky (equity) returns.

The Idea of an Internal Swap Market

The traders in this swap market are young and old individuals who are saving for retirement income via DC plans. This internal market can be organized for workers who are covered by the same terms of employment including the pension plan, (e.g. a specific group of professionals such as physicians or lawyers). We develop a life-cycle model to determine the risk premiums for different cohorts and consequently the optimal age-related exposure to such swaps, in addition to the conventional market portfolio of equities and bonds.

We show that the internal market of wage-linked swaps can be an ideal construction to achieve intergenerational risk sharing between young and old plan members, and simultaneously to create optimal life-cycle exposures for all members. It also helps to complete the missing market of wage-linked securities, and to relax the borrowing constraints of young members. The largest welfare gains are achieved if employers require mandatory participation in these swap contracts over long periods (e.g. throughout one's career).

The remainder of this article describes a pension plan design with deferred variable annuities. From there, it investigates whether or not the participants of different ages have incentives to engage in a return-wage swap arrangement. The conclusion is that younger and older workers may prefer different levels of return-wage swaps, therefore creating trading possibilities for welfare improvement. A pricing protocol is developed which converts the incentives into the fair price of such a swap and the market clearing condition. The article concludes that this swap protocol could be the basis for a win-win outcome for both younger and older workers.

Pension Plan Design

We analyze a pension plan with a deferred variable annuity structure that is offered to a group of employees of different ages. The plan members contribute a fraction of their wage income into the plan for every year of service. Contributions are used to acquire deferred variable annuities paying out during retirement. If the contributions are invested only in (nominal) risk-free assets, then effectively each unit of contribution buys a certain quantity of deferred annuity at the market nominal rate of interest.

Let $P_{t,x}$ stand for the contribution made at time t when a plan member is of age x , R^f for the nominal risk-free rate, and ${}_k p_x$ the survival probability. Then the equation below relates the contribution to the present value of a deferred annuity with a payment level of $\overline{\Delta B}_t$:

$$P_{t,x} = \sum_{k=65-x}^{100-x} \left(\exp(-kR_k^f) ({}_k p_x) \overline{\Delta B}_t \right) \quad (1)$$

When the contributions for each year are invested in nominal riskfree assets, then income upon retirement is equal to the fixed amount of $\sum_{25}^{64} (\overline{\Delta B}_t)$.

In practice, the assets backing these acquired deferred annuities are often invested in the risky assets, so that the participants have prospect on earning the (equity) risk premiums, given their risk appetite. We capture this by adjusting yearly the benefits from the variable annuities for the excess return of realized investment return over the risk-free rate. Let $\Delta \tilde{B}_{t,T}$ reflect the benefits with adjustments, at retirement T the bought deferred annuity in year t , $\overline{\Delta B}_t$ amounts to:

$$\Delta \tilde{B}_{t,T} = \overline{\Delta B}_t \exp \left(\sum_{s=t}^T (R_s^P - R_s^f) \right), \quad (2)$$

where R_s^P is the return on a broadly diversified market portfolio of stocks and nominal bonds.

Upon retirement, the total benefit paid out is:

$$B_T = \sum_{t=0}^T \overline{\Delta B}_t \exp \left(\sum_{s=t}^T (R_s^P - R_s^f) \right), \quad (T > t) \quad (3)$$

In term of accumulated assets upon retirement, equation (3) is equivalent to the following expression:

$$A_T = \sum_{t=0}^T P_{t,x} \exp\left(\sum_{s=t}^T R_s^P\right) \quad (4)$$

Internal Swap Incentives

The collective portfolio of stocks and bonds may not be the optimal portfolio when one takes into account wage growth uncertainty. Life-cycle theory indicates that the total wealth of the younger cohorts consists primarily of human capital, whereas the older cohorts have a relatively large position in financial capital and a decreasing value of human capital. Adding wage-linked assets to the portfolio may be better from the perspective of risk diversification and wage inflation protection.

Young and old likely have opposite preferences as to wage-linked assets. The younger cohorts will probably prefer a short position in wage-linked assets as they are overexposed to wage growth risk, whereas the older cohorts will look for a long position in wage-linked assets to offset the high exposure to financial market risk. However the supply of the price inflation-linked assets is limited in financial markets and wage-linked securities are not yet available at all.

As younger workers are natural suppliers of wage-linked securities, older workers are natural demanders of these assets. In the next section we test whether young and old employees have incentives to trade wage-linked return for capital market return. We construct an internal market which is accessible to employees covered by the plan and these individuals trade wage-linked swaps.

We investigate the preferred wage-linked positions by the various cohorts with help of the expression below, wherein α_x is the preferred holding of financial wealth in the available portfolio of stocks and bonds, the complement $1-\alpha_x$ the preferred holding in wage-linked assets, and w_s denotes the rate of wage inflation.

$$B_T = \sum_{t=0}^T \overline{\Delta B}_t \exp\left(\sum_{s=t}^T (\alpha_x (R_s^P - R_s^f) + (1-\alpha_x)w_s)\right) \quad (5)$$

Note when $\alpha_x=1$, then the adjustment in the value in the deferred annuities is purely driven by financial returns and when $\alpha_x=0$, then the adjustments are purely linked to wage growth.

Accumulated assets upon retirement, solved from equation (5), can be expressed as follows:

$$A_T = \sum_{t=0}^T P_{t,x} \exp\left(\sum_{s=t}^T (R_s^f + \alpha_x (R_s^p - R_s^f) + (1 - \alpha_x)w_s)\right) \quad (6)$$

For the younger cohorts it might be that the preferred α is larger than one, $\alpha_x > 1$, meaning they wish to acquire extra financial returns by shorting the wage growth exposure. In order to explore the existence of incentives, we use a welfare measure, which is defined by the preferences of individuals.

Employee Preferences

The preference of the employee is defined by the expected utility of wage-deflated income. That is, individuals evaluate retirement income in a real framework deflated by wage growth. The utility follows CRRA preference, i.e., $u(X) = X^{1-\gamma} / (1-\gamma)$, where gamma is the relative risk aversion, and X is the income level. During the working period, net-income is gross wage minus pension contributions ($Y_n^x - P_v$). Upon retirement, the accumulated wealth is A_{65} . If annuitized at nominal risk free rate, then the retirement income is equal to the payout of the benefits (B_{65}).

$$U_{x \in [25, 64]} = E_0 \left[\sum_{t=0}^{64-x} e^{-\beta t} u\left(\frac{Y_t - P_t}{W_t}\right) \right] + E_0 \left[e^{-\beta*(65-x)} u\left(\frac{A_{65}}{W_{65}}\right) \right] \quad (7)$$

where W_t denotes wage inflation index upon retirement: $W_T = \exp\left(\sum_{k=t}^T w_k\right)$

Since the choice of the values of α has no impact on the before-retirement net-income, the preference can be simplified to the expected utility over retirement income. The preference is equivalent to:

$$U_{x \in [25, 64]} = E_0 \left[e^{-\beta*(65-x)} u\left(\frac{A_{65}}{W_{65}}\right) \right],$$

which is also equivalent to if annuitized at risk free:

$$U_{x \in [25, 64]} = E_0 \left[e^{-\beta^*(65-x)} u \left(\frac{B_{65}}{W_{65}} \right) \right] \quad (8)$$

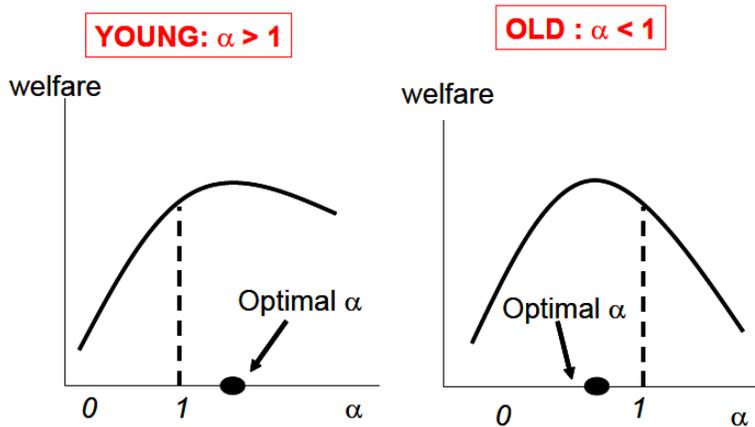
The welfare level is measured by real certainty equivalent benefit (CEB). For employees, it is calculated by solving the following equation:

$$U_{x \in [25, 64]} = e^{-\beta n} u(CEB^x) \quad (9)$$

$$\text{Hence, } u(CEB^x) = \frac{CEB^{1-\gamma}}{1-\gamma} \Rightarrow CEB = \left((1-\gamma) \frac{U}{e^{-\beta n}} \right)^{\frac{1}{1-\gamma}} \quad (10)$$

The illustrative CEB- α curve below illustrates the incentives of different age cohorts to swap. Younger workers do hold variable annuities in market portfolio ($\alpha=1$) but they might prefer more of them ($\alpha>1$). That is, they want to sell wage growth risk in order to buy additional asset returns. At the same time, older workers also hold variable annuities in market portfolio ($\alpha=1$), but they might prefer to hold less ($\alpha<1$). Thus they want to buy wage growth risk and to sell asset return.

Figure 1: Incentives of Swapping: Young and Old Members' Perspectives



Actual Results

The results discussed below assume an economy with an equity premium of 5%, average nominal risk-free rate of 3%, average portfolio (60% in equities) excess return of 3%, and average wage growth of 2%. The volatility of equity returns is set at 15% per year and wage

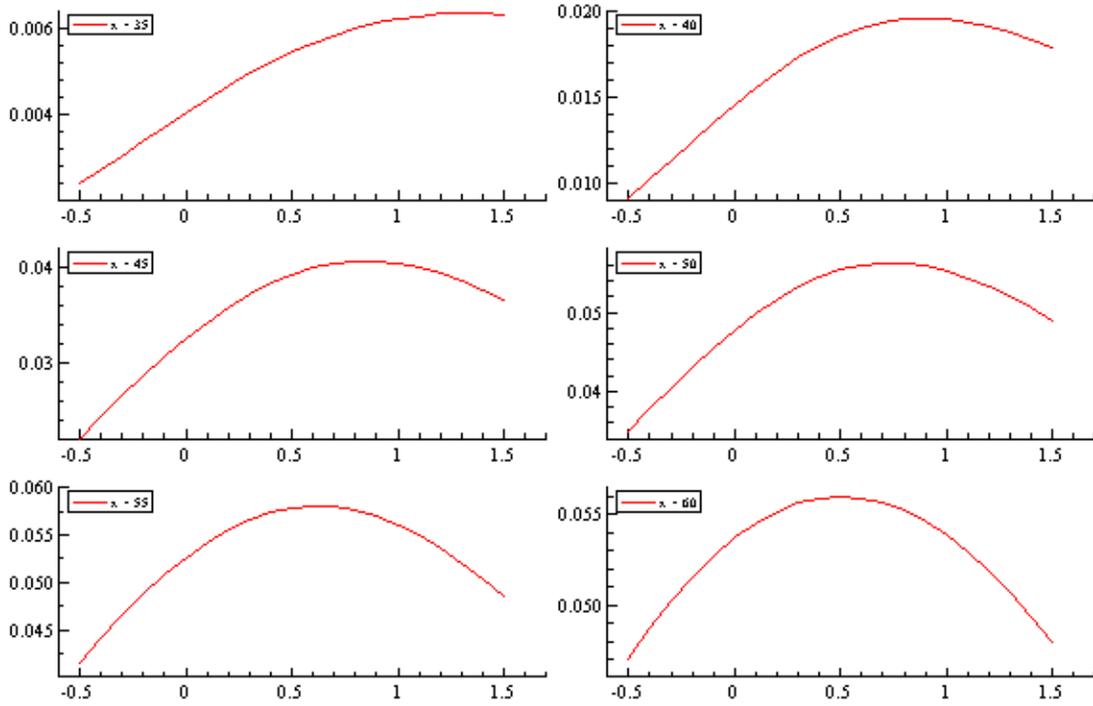
growth volatility at 1.5% per year⁵. The correlation between wage growth and equity returns is a small but positive 0.1. Under these assumptions, do participants have different preferences regarding their exposure to financial returns and wage growth? If so, do they have incentive to swap the financial returns and wage growth exposures with each other?

Figure 2 shows the different return exposures and the calculated welfare improvements for employees ages 35 to 60. The CEB- α curves are hump shaped. We see clearly the most preferred return exposure of the young is larger than that of the older cohorts. For instance, the 35-year-old prefers $\alpha=130\%$ exposure to return indexation and hence negative 30% in wage indexation. The 60-year-old and retirees prefer about 50% exposure to return indexation and 50% in wage indexation. This result is consistent with the life-cycle theory: younger cohorts prefer to be overexposed to financial returns because of their human capital. This can be seen from the benefit equation (4). Part of young employees' benefit accruals come from the to-be-earned salary incomes, and therefore have a large position in wage growth and wage risk.

Older workers have a relatively small position in wage growth exposure compared to their financial wealth and they may benefit from selling return risk in order to increase wage growth return and risk exposure. For example, the 60-year old individual prefers a portfolio of 50% in financial return exposure and 50% in wage return exposure. These figures indicate that it is possible to improve the exposures of different cohorts by setting up an internal swap market, which results in age-dependent indexation policy.

⁵ The equity returns and wage growth are i.i.d.

Figure 2: The CEB- α Curve for Employees of Age 35, 40, 45, 50, 55 and 60*



* With assumed risk aversion $\gamma=10$. The maturity of the swaps equals the remaining working life ($n=65-x$).

A Market-Clearing Mechanism

Although the above results are intuitive, a drawback is that the α parameter is fixed during the remaining working period of the young. With this condition, in a certain period of time, all future cohorts and current young members would demand more financial returns, but there will be little or no supply. Therefore, a scheme with fixed α is not sustainable. Thus we must make α age-dependent: making it decline with age. Our previous results will still hold in that there will still be incentives to swap. We now incorporate a market clearing condition to equalize the supply and demand.

$$\sum_{x \in \{\alpha_x > 1\}} L_x N_x (\alpha_x - 1) w_t = \sum_{x \in \{\alpha_x < 1\}} L_x N_x (1 - \alpha_x) w_t \quad (11)$$

where L_x denotes the asset under management per age cohort, and N_x denotes the population size of each cohort. Here we study a stationary situation where the cohorts are of equal size.

With the market clearing condition, the sum of the total wage-linked claims offered by the younger cohorts equal the total wage-linked claims demanded by the older cohorts, in each period for the coexisting working participants.

However, are there still incentives to swap? Consider the case, for example, before market is cleared, there is more demand for wage-linked claims from the older cohorts than the total supply from the younger cohorts. To clear the market, younger workers must be compensated to motivate them to provide more wage-linked claims in exchange of extra financial risks. There are several ways of providing such compensation. One possibility is that the net suppliers invest the cash compensation into risky assets in order to boost their future retirement benefit payments.⁶ We have shown above that, if wage-linked claims were available, the benefits could be:

$$B_T = \sum_{t=0}^T \overline{\Delta B_t} \exp\left(\sum_{s=t}^T (\alpha_x (R_s^p - R_s^f) + (1 - \alpha_x) w_s)\right) \quad (12)$$

With positive compensation per unit of risk taking ($c > 0$) to boost future benefit payments, the risk exposure can be modified as follows:

$$B_T = \sum_{t=0}^T \overline{\Delta B_t} \exp\left(\sum_{s=t}^T (\alpha_x (R_s^p - R_s^f) + (1 - \alpha_x) w_s + (\alpha_x - 1)c)\right) \quad (13)$$

When $\alpha_x - 1 > 0$ the person acts as a net supplier and is compensated, and when $\alpha_x - 1 < 0$ the person is a net demander and pays for it.

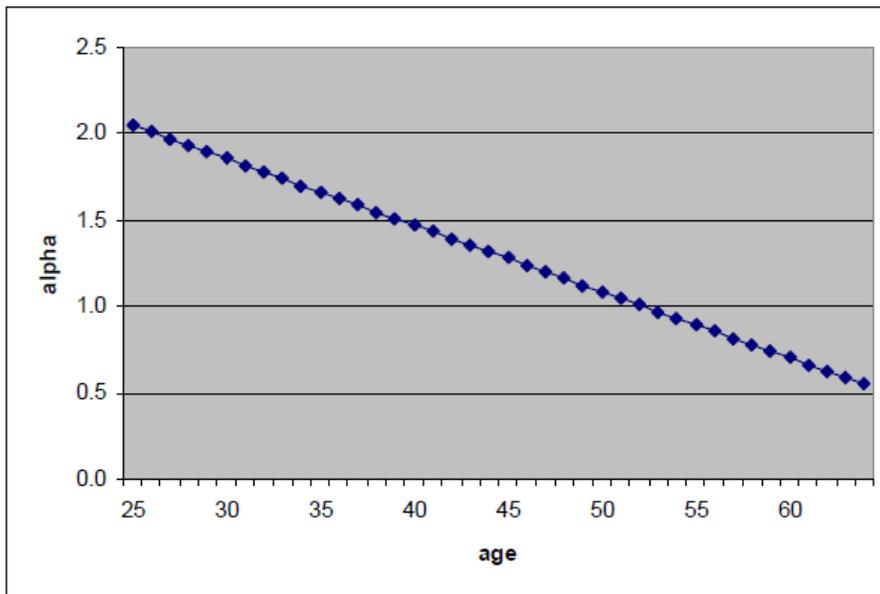
The compensation, c , is determined jointly with the shape of α_x . The procedure is the following: first, a set of α_x which satisfies the market clearing equation (11) are determined. Then, we evaluate the benefit levels with a set of values for c as in (12) using the welfare measure (expression (8) and (10)). We choose the level of c and α_x so that the welfare level of each generation is higher than, or equal to, the case with only financial exposures (i.e. $\alpha_x = 1$).

⁶ Another possibility is that the *net demanders* (e.g. older cohorts) pay a certain amount of cash to the *net suppliers* (e.g. the younger cohorts) up front for the future wage-linked claims. The net suppliers consume the cash compensation immediately.

Findings

Figure 3 shows the pattern of α_x , which decreases with age. Prior to age 52, workers are suppliers of wage-linked claims and after age 52, they are receivers of wage-linked claims. The magnitude of the compensation is approximately 0.5 basis points per one percentage point increase of extra risk taking.

Figure 3: Degree of Swap



The resulting pay-off scheme is summarized in Table 1. Because of the decline of α_x with age, the expected total excess return from the variable annuity also declines with age. After age 52, the expected excess return is lower than the excess return of the financial assets. However, the older employees receive the desired less volatile wage growth. The last column shows the average supply (-) and demand (+) of wage-linked claims per age cohort, relative to the size of the total assets under management. Each year, the variable annuity participants have the opportunity to renegotiate whether to extend the internal swap for the next year.⁷

⁷ The internal market might break down if certain cohorts do not have sufficient incentive to participate (e.g. due to mismatch between supply and demand, varying sizes of cohorts, etc).

Table 1: The Pay-off Scheme (correlation = 0.1)

age	α	$\alpha^* E[R^e]$	$(1-\alpha)E[w]$	$(\alpha-1)*c$	E[return]	supply(-) or demand(+)
25	2.1	6.3%	-2.2%	0.55%	4.7%	0.001%
35	1.7	5.1%	-1.4%	0.35%	4.1%	0.015%
45	1.3	3.9%	-0.6%	0.15%	3.5%	0.014%
55	0.9	2.7%	0.2%	-0.05%	2.9%	-0.008%
60	0.7	2.1%	0.6%	-0.15%	2.6%	-0.029%

If the internal swap is able to continue operating, then the resulting welfare improvements for some age cohorts, relative to the welfare levels obtained without any swap (i.e., $\alpha=1$ in Figure 2), are shown in Table 2. With appropriate compensation for risk, the internal swap market improves the welfare of the participants, especially the younger members. The older cohorts reach break-even at a reduced level of demand for wage-linked claims. These welfare gains could be reached through mandatory participation in the swap contract over extended periods of time (e.g. throughout workers' careers).

Table 2: Welfare Improvements at Implementation of a Swap Market with Continued Operation (correlation = 0.1)

age	35	40	45	50	55	60
change in welfare at implementation	9.2%	4%	2.5%	1.1%	0.4%	0%

Sensitivity analysis

This session we show how sensitive is the internal swap arrangement sensitive to the correlation between equity market and wage growth rate. In the baseline model, a small but positive correlation of 0.1 is assumed. However, when the correlation gets larger, the benefit of diversification is reduced. Hence, for the younger cohorts, it leads to less demand of financial returns and less incentive of supplying wage growth exposure to the old cohorts. We recalculate the α_x curve and the compensation c , under the situation with a higher correlation of 0.3 between equity return and wage growth rate. The new α_x curve is flatter than that in the baseline case (Figure 4), and the required compensation is higher at $c = 0.9$ basis points per one

percentage point increase of extra risk taking. The welfare gains due to the swap are still positive, but less (Table 3). This is because a higher correlation reduces the incentives of swapping and leads to an increase in the required compensation c . The new payoff scheme is shown in Table 4.

Figure 4 Degree of Swap

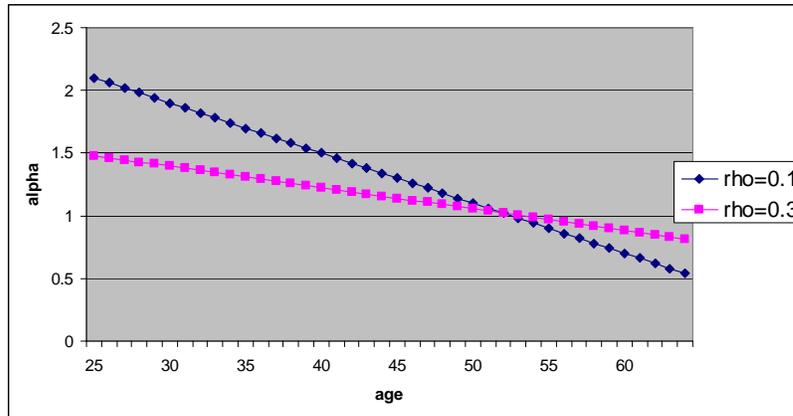


Table 3: Welfare Improvements at Implementation of a Swap Market with Continued Operation (correlation = 0.3)

age	35	40	45	50	55	60
change in welfare at implementation	3.2%	1.4%	0.9%	0.4%	0.1%	0.0%

Table 4: The Pay-off Scheme

age	α	$\alpha^* E[Re]$	$(1-\alpha)E[w]$	$(\alpha-1)*c$	$E[\text{return}]$	supply or demand
25	1.48	4.4%	-1.0%	0.43%	3.9%	0.000%
35	1.31	3.9%	-0.6%	0.28%	3.6%	0.007%
45	1.14	3.4%	-0.3%	0.13%	3.3%	0.007%
55	0.97	2.9%	0.1%	-0.03%	2.9%	-0.002%
60	0.89	2.7%	0.2%	-0.10%	2.8%	-0.011%

A Win-Win Solution

There is a global trend to substitute funded DC pension plans for price-indexed and wage-indexed DB plans. However participants in DC plans have limited, or no access to inflation-linked assets, especially wage inflation-linked claims. But different age cohorts have different exposure to the wage growth potential captured in their human capital position. This creates the opportunity to trade wage growth for financial returns.

We show that the creation of an internal swap market helps to overcome the market incompleteness regarding wage-linked securities, and hence improves welfare for DC scheme members. We find the young are able to offset their high exposure to wage growth and increase their exposure to capital market return, and older workers replace full exposure to financial markets by a combination of wage and capital market returns. This reallocation of risk exposures over the ages is in line with the recommendations of life-cycle theory of personal finance.

References

- Bodie, Z., McLeavey, D., and Siegel, L.B. (2007). *The future of life-cycle saving and investing*. Research Foundation of CFA Institute.
- Bovenberg, L., Koijen, R., Nijman, T., and Teulings, C. (2007). Saving and investing over the life-cycle and the role of collective pension funds. *De Economist*, 155(4). [doi:10.1007/s10645-007-9070-1](https://doi.org/10.1007/s10645-007-9070-1).
- Campbell, J.Y. and Viceira, L. (2002). *Strategic asset allocation: Portfolio choice for long-term investors*. Oxford University Press.
- Choi, J.J., Laibson, D., and Madrian, B.C. (2004). *Plan design and 401(k) saving outcomes*. NBER Working Paper 10486. [doi:10.3386/w10486](https://doi.org/10.3386/w10486).
- Cui, J., De Jong, F., and Ponds, E. (2011). Intergenerational risk sharing within funded pension schemes. *Journal of Pension Economics and Finance*, 10 (1) : 1-29, January, 2011. <http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=7947375>
- Davidoff, T., Brown, J., and Diamond, P. 2005. Annuities and Individual Welfare. *American Economic Review*. 95(5), 1573-1590. [doi:10.1257/000282805775014281](https://doi.org/10.1257/000282805775014281).
- De Jong, F. (2008), Valuation of Pension Liabilities in Incomplete Markets. *Journal of Pension Economics and Finance*. 7, 277-294. [doi:10.1017/S1474747208003673](https://doi.org/10.1017/S1474747208003673).

Gollier, C. (2008). Intergenerational risk sharing and risk taking of a pension fund. *Journal of Public Economics*. 92(5-6). 1463-1485. [doi:10.1016/j.jpubeco.2007.07.008](https://doi.org/10.1016/j.jpubeco.2007.07.008).

Matsen, E., and Thogerson, O. (2004). Designing social security - A portfolio approach. *European Economic Review*. 48. 883-904. [doi:10.1016/j.euroecorev.2003.09.006](https://doi.org/10.1016/j.euroecorev.2003.09.006).

Molenaar, R. and Ponds, E. (2010). Risk sharing and individual lifecycle investing in funded collective pensions. *Journal of Risk* (forthcoming)

Teulings, C., and De Vries, C. (2006). Generational accounting, solidarity, and pension losses. *De Economist*, 154(1), 63-83. [doi:10.1007/s10645-006-6486-y](https://doi.org/10.1007/s10645-006-6486-y).

Viceira, L. (2008). *Life-cycle funds - Overcoming the saving slump: How to increase the effectiveness of financial education and saving programs*. Annamaria Lusardi, ed., University of Chicago Press.

Yaari M. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies* .32(2). [doi:10.2307/2296058](https://doi.org/10.2307/2296058).